Math 304 (Spring 2015) - Homework 4

Please note that this homework has two pages. There are 5 problems in total.

Problem 1.

Determine whether the following sets are subspaces of $\mathbb{R}^2$. Explain why!

(1) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid x_1 + x_2 = 0 \right\}$

(2) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid x_1 x_2 = 0 \right\}$

(3) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid x_1 + x_2 = 1 \right\}$

(4) $\left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mid x_1^2 + x_2^2 = 1 \right\}$

Problem 2.

Let us denote by $M_{2\times 2}(\mathbb{R})$ the set of all (2×2) matrices whose entries are all real numbers. In the textbook, $M_{2\times 2}(\mathbb{R})$ is also denoted by $\mathbb{R}^{2\times 2}$. We know that $M_{2\times 2}(\mathbb{R})$ is a vector space. Now determine whether the following sets are subspaces of $M_{2\times 2}(\mathbb{R})$. Explain why!

(1) The set of all (2×2) upper triangular matrices.

(2) The set of all (2×2) nonsingular matrices.

(3) The set of all (2×2) symmetric matrices.

(4) The set of all (2×2) matrices with determinant equal to 1.

Problem 3.

(a) Let $\mathbb{P}_3$ be the vector space of all polynomials with degree less than or equal to 3. Determine whether the following sets are subspaces of $\mathbb{P}_3$.

  (1) The set of all polynomials $p(x)$ in $\mathbb{P}_3$ such that $p(0) = 0$.

  (2) The set of all polynomials $p(x)$ in $\mathbb{P}_3$ such that $p(0) = 1$.

(b) Let $C[-\pi, \pi]$ be the vector space of all continuous functions on the closed interval $[-\pi, \pi]$. Determine whether the following sets are subspaces of $C[-\pi, \pi]$.

  (1) The set of all *odd* functions in $C[-\pi, \pi]$.

  (2) The set of functions $f(x)$ in $C[-\pi, \pi]$ such that $f(-\pi) = f(\pi)$.

Problem 4.

Determine whether the following vectors form a spanning set of $\mathbb{R}^3$.

$$\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

Problem 5.

Find the null space of the matrix

$$A = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 2 & -3 & 1 \\ -1 & -1 & 0 & -5 \end{pmatrix}$$